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STABILITY OF A TRANSLATING BEAM WITH FIXED AND MOVING VISCOUS DAMPING

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1. Introduction

The vibration and stability of moving media, such as magnetic tapes in computer systems, are often analyzed by using a translating string as a model. The dynamics of such a model is similar but not identical to that of a translating Bernoulli-Euler beam, which is a more accurate model of systems with flexural rigidity. The stability of these kinds of systems has been investigated by various authors in recent years: Crandall [1], Wickert & Mote [2], Triantafyllou [3], and Cheng & Perkins [4], among others. In this note, the influence of moving and stationary damping coefficients on the stability of the moving elastic medium is investigated. The notation here follows that of Crandall [1].

2. Equations of motion

The moving medium under analysis is assumed to have the following parameters: stiffness, EI; axial force, T; elastic foundation stiffness, α; uniform translational velocity, v; and stationary and moving viscous damping coefficients, b_s and b_m respectively. The governing equation of motion in a fixed frame of reference is, after Crandall [1],

$$EI\frac{\partial^4 w}{\partial x^4} - T\frac{\partial^2 w}{\partial x^2} + \alpha w + b_s\frac{\partial w}{\partial t} + b_m\frac{Dw}{Dt} + \rho A\frac{D^2 w}{Dt^2} = 0. \tag{1}$$

The convective term derivative may be defined in terms of the convective operator, $\mathbf{D}$, as

$$\mathbf{D} = \frac{D}{Dt} = \frac{\partial}{\partial t} + v\frac{\partial}{\partial x} \quad \Rightarrow \quad \mathbf{D}^2 = \left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)^2. \tag{2}$$

In solving this problem as a stability problem, it is convenient to use a wave propagation approach. Assuming that a wave having a spatial wave number κ propagates along the beam at a vibration frequency ω, one may write $w = w(x, t) = w_0\,e^{i(\kappa x - \omega t)}$, where w_0 is the vibration amplitude, and i is the unit imaginary number. The existence of a differentiable mapping, $\mathbf{f}$: $\{\mathscr{L}_i \in \mathbb{R}^2\} \rightarrow s \in \mathbb{C}$, carrying linear operators $\mathscr{L}_i$ from $\mathbb{R} \times \mathbb{R}$ to $\mathbb{C}$, is evident. Functions which are elements of the map $\mathbf{f}$ are

$$f_1\colon \frac{\partial}{\partial x} \rightarrow i\kappa, \quad f_2\colon \frac{\partial^2}{\partial x^2} \rightarrow -\kappa^2, \quad f_3\colon \frac{\partial^4}{\partial x^4} \rightarrow \kappa^4, \quad f_4\colon \frac{\partial}{\partial t} \rightarrow -i\omega,$$

$$f_5\colon \frac{\partial^2}{\partial t^2} \rightarrow -\omega^2, \quad f_6\colon \mathbf{D} \rightarrow -i(\omega - \kappa v), \quad f_7\colon \mathbf{D}^2 \rightarrow -(\omega - \kappa v), \tag{3}$$

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3. STABILITY ANALYSIS

A central issue in the dynamics of moving media is the computation of stability boundaries. Closed form solutions for the desired stability boundaries are readily effected by making use of certain relatively new ideas from algebraic geometry, such as transversality, and singularities of plane algebraic curves. These ideas have been introduced to engineering analysis by Afolabi [5, 6], and the first step in their engineering application is to derive the characteristic polynomial of the vibrating system.

From equations (1)–(3), one arrives at the characteristic polynomial of the present system as

$$p(w; v) = EI\kappa^4 + T\kappa^2 + \alpha - \mathrm{i}\omega b_s - \mathrm{i}(\omega - \kappa v)b_m - \rho A(\omega - \kappa v)^2 = 0. \tag{4}$$

In the above, ω is the indeterminate, whilst v may be regarded as the "control parameter". The concept of a control parameter is important when making catastrope-theoretic analyses of stability; see, for instance, Afolabi [5]. By recasting equation (4), one obtains a quadratic polynomial with complex coefficients:

$$p(\omega; v) = a_0\omega^2 + a_1\omega + a_2 = 0, \tag{5}$$

where

$$a_0 = \rho A, \quad a_1 = -2\rho A\kappa v + \mathrm{i}(b_s + b_m), \quad a_2 = \rho A\kappa^2 v^2 - (EI\kappa^4 + T\kappa^4 + \alpha) - \mathrm{i}\kappa v b_m.$$

3.1. *Divergence instability*

The criterion for finding the divergence boundaries is this: *if there exists a divergence instability*, *then the constant term in the characteristic polynomial* (5) *must vanish at the divergence boundary*; see, for instance, reference [7]. Only real values of v are admissible solutions in this requirement. Separating a_2 into its real and imaginary components yields $a_2(v, \kappa) = a_{2r}(v, \kappa) + \mathrm{i}a_{2i}(v, \kappa) = 0$, where

$$a_{2r}(v, \kappa) = \rho A\kappa^2 v^2 - EI\kappa^4 - T\kappa^2 - \alpha = 0, \quad a_{2i}(v, \kappa) = \kappa v b_m = 0. \tag{6}$$

Since ρA, EI, T, and α do not vanish in the non-trivial case, it is evident from equation (6) that the principal parameter which inhibits the onset of divergence instabilities is the *moving* viscous damping coefficient, b_m. Thus, if $b_m \neq 0$, *then divergence cannot occur*. However, if $b_m = 0$ a critical speed of divergence, v_D^*, is encountered when

$$v = v_D^* = \sqrt{(EI\kappa^4 + T\kappa^2 + \alpha)/\rho A\kappa^2} \tag{7}$$

From the foregoing, one reaches a well-known conclusion that a positive damping coefficient $b_m > 0$ has a beneficial effect on the system by stabilizing an otherwise divergence instability. It is remarkable that a *negative* damping coefficient ($b_m < 0$) also has a stabilizing effect with respect to divergence instability, a characteristic that is well known of gyroscopic systems. The primary condition governing the inhibition of divergence is this: *the constant term of the characteristic polynomial must be a complex number with a non-zero imaginary part*. It is immaterial whether the imaginary part is positive or negative. This is, however, not the case for flutter, where the sign of the moving damping coefficient is significant.

3.2. *Flutter instability*

The flutter criterion is based on the consideration that the root locus of the vibrating system in the Argand or complex s-plane intersects the imaginary axis transversally. Consequent on this assumption, it is easy to prove the flutter criterion, namely: *at a flutter boundary*, *the real and imaginary parts of the characteristic polynomial must vanish* *simultaneously*; see, for instance reference [8]. By means of this criterion, one obtains the critical flutter velocity, v_F^*, as

$$v = v_F^* = (1 + b_m/b_s)\sqrt{(EI\kappa^4 + T\kappa^2 + \alpha)/\rho A\kappa^2}. \tag{8}$$

For a string having a negligible flexural stiffness, one merely sets $EI = 0$ in equation (8) to obtain the corresponding expression for the flutter velocity.

4. DISCUSSION AND CONCLUSIONS

The effect of moving and stationary viscous damping on stability may now be investigated. When the moving viscous damping is made to vanish by setting $b_m = 0$, $b_s \neq 0$ in equation (8), it becomes evident that flutter instability is not inhibited to any great extent by adding or deleting moving viscous damping. This may be contrasted with the case of divergence instability, which is inhibited considerably when moving viscous damping is added. Moreover, unlike in the case of divergence instability where negative moving damping has a stabilizing effect, a negative b_m evidently has an adverse effect on stability, except when the stationary damping is simultaneously negative.

In the second scenario, if the stationary viscous damping is made to vanish completely, i.e. $b_s = 0$, $b_m \neq 0$, then one comes to the remarkable result that the onset of flutter is completely inhibited. The smaller the stationary viscous damping gets, the further the flutter speed is pushed beyond normal operating speeds. The same effect may be achieved by an increase in b_m and a simultaneous decrease in b_s. However, the dominant factor in the inhibition of flutter is the reduction of *stationary* viscous damping.

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